Linear and non-linear parabolic equations: weak formulations, approximations and analysis (2018)
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Lecture Structure

1 Preliminaries (Chapter 5 in [10]).

- Sobolev spaces.
- Inequalities.
- Bochner spaces.
- Compactness arguments. Papers: [31].

2 Linear parabolic equations (Chapter 7 in [10] and Chapter 6 in [10]).

- Weak formulation (continuous variational fromulation), regularity of solution.
- Semigroup theory.
- Rothe method, backward-Euler - Galerkin FE approximations/MFEM.
- Stability and error estimates (Galerkin FE and MFEM).

3 Non-linear parabolic equations (different sources)

- Weak solutions, regularity.
- Fixed point theorems (Chapter 9 in [10]).
- Stability and error estimates (Galerkin FE and MFEM). Papers: [3, 2, 28, 5].
- Linearization methods.

4 Degenerate parabolic equations (different sources).

- Weak formulation (continuous variational fromulation), regularity of solution. Papers: [1, 27].
- Numerical approximations (Galerkin FE and MFEM).
- Stability and error estimates, Kirchhoff's transformation and Green's operator. Papers: [18, 19, 23, 27, 21].
- Linearization methods. Papers: [20, 15, 24]
- Domain decomposition methods. Papers: [29, 30]

5 Coupled elliptic and parabolic equations (different sources).

- Reactive transport in porous media [13, 14, 17, 22, 26, 25].
- Poromechanics (Biot's model). Papers: [11, 16, 8, 4, 6, 7].

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